Subset Space Public Announcement Logic

Corrections and improvements

March 21, 2018

1. The axiom Cl should be replace with a rule "from $\varphi \leftrightarrow \psi$ infer $\mathsf{pre}(\varphi) \leftrightarrow \mathsf{pre}(\psi)$", which is known as "replacement of equivalents".
2. In the axiomatization **PAL** (Figure 4), add a new reduction axiom

$$[\varphi]\mathsf{pre}(\psi) \leftrightarrow (\mathsf{pre}(\varphi) \to \mathsf{pre}(\varphi \wedge [\varphi]\psi))$$

Otherwise the proof system is not reducible to $\mathbf{EL}^+$. We thank Philippe Balbiani for raising this question. We give a proof of the validity of this reduction axiom.

Proof. First of all,

$$\begin{array}{ll} & \mathcal{X}, x, O \models [\varphi]\mathsf{pre}(\psi) \\ \text{iff} & x \in (\!|\varphi|\!)^O \in \mathcal{O} \Rightarrow \mathcal{X}, x, (\!|\varphi|\!)^O \models \mathsf{pre}(\psi) \\ \text{iff} & x \in (\!|\varphi|\!)^O \in \mathcal{O} \Rightarrow x \in (\!|\psi|\!)^{(\!|\varphi|\!)^O} \in \mathcal{O} \end{array}$$

On the other hand,

$$\begin{array}{ll} & \mathcal{X}, x, O \models \mathsf{pre}(\varphi) \to \mathsf{pre}(\varphi \wedge [\varphi]\psi) \\ \text{iff} & \mathcal{X}, x, O \models \mathsf{pre}(\varphi) \Rightarrow \mathcal{X}, x, O \models \mathsf{pre}(\varphi \wedge [\varphi]\psi) \\ \text{iff} & x \in (\!|\varphi|\!)^O \in \mathcal{O} \Rightarrow x \in (\!|\varphi \wedge [\varphi]\psi|\!)^O \in \mathcal{O} \end{array}$$

It suffices to show that $(\!|\psi|\!)^{(\!|\varphi|\!)^O} = (\!|\varphi \wedge [\varphi]\psi|\!)^O$ under the condition $(\!|\varphi|\!)^O \in \mathcal{O}$. Now,

$$(\!|\psi|\!)^{(\!|\varphi|\!)^O} = \{y \in (\!|\varphi|\!)^O \mid \mathcal{X}, y, (\!|\varphi|\!)^O \models \psi\}$$

and

$$\begin{array}{rcl} (\!|\varphi \wedge [\varphi]\psi|\!)^O & = & \{y \in O \mid \mathcal{X}, y, O \models \varphi \wedge [\varphi]\psi\} \\ & = & \{y \in O \mid \mathcal{X}, y, O \models \varphi \;\;\&\;\; \mathcal{X}, y, O \models [\varphi]\psi\} \\ & = & \{y \in O \mid y \in (\!|\varphi|\!)^O \;\;\&\;\; (y \in (\!|\varphi|\!)^O \in \mathcal{O} \Rightarrow \mathcal{X}, y, (\!|\varphi|\!)^O \models \psi)\} \\ & = & \{y \in (\!|\varphi|\!)^O \mid (\!|\varphi|\!)^O \in \mathcal{O} \Rightarrow \mathcal{X}, y, (\!|\varphi|\!)^O \models \psi\} \\ & = & \{y \in (\!|\varphi|\!)^O \mid \mathcal{X}, y, (\!|\varphi|\!)^O \models \psi\} \text{ under the condition } [\![\varphi]\!]^O \in \mathcal{O} \end{array}$$